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$$= \sqrt{\{[a^2 + 2\sqrt{abc}][b^2 + 2\sqrt{abc}][c^2 + 2\sqrt{abc}]\}}.$$

This gives us the values of the sides.

Otherwise draw AO and produce AO to O_1 so that $OO_1 = 2\sqrt{(bc/a)}$. Upon OO_1 as diameter describe a circle. With O as a center and b as a radius describe an arc cutting the circle in B . Similarly, with O as center and c as radius, draw an arc cutting the circle in C . Join BC , AC , AB , then ABC is the triangle required. For O_1 is the ex-center opposite A by construction as follows :

$$AO_1 = p/\cos \frac{1}{2}A. \quad \therefore AOA O_1 = yz = AO^2 + 2\sqrt{(AO \cdot BO \cdot CO)}.$$

$$\therefore AO_1 = AO + 2\sqrt{[(BO \cdot CO)/AO]}.$$

CALCULUS.

121. Proposed by W. W. LANDIS, A. M., Professor of Mathematics and Astronomy, Dickinson College, Carlisle, Pa.

$$\text{Solve the differential equation } \left[\frac{d}{dx} + b \right]^n y = \cos ax.$$

Solution by LON C. WALKER, A. M., Petaluma High School, Petaluma, Cal., and LEWIS NEIKIRK, B. S., Boulder, Col.

$$\left[\frac{d}{dx} + b \right]^n y = \cos ax.$$

$$\left[\frac{d}{dx} + b \right]^n \text{ has } n \text{ roots each } = -b.$$

$$\therefore \text{Comp. Factor} = e^{-bx}(c_1 + c_2x + c_3x^2 + c_4x^3 + \dots c_nx^{n-1}).$$

$$\frac{1}{\left[\frac{d}{dx} + b \right]^n} \cos ax = \frac{\left[\frac{d}{dx} - b \right]^n}{\left[\frac{d}{dx} - b^2 \right]^n} \cos ax = \left\{ \frac{b - \frac{d}{dx}}{a^2 + b^2} \right\}^n \cos ax.$$

$$\text{Let } n=1. \quad \therefore \frac{b - d/dx}{a^2 + b^2} \cos ax = \frac{1}{a^2 + b^2} (b \cos ax + a \sin ax)$$

$$= \frac{1}{(a^2 + b^2)^{\frac{1}{2}}} \left[\frac{b \cos ax + a \sin ax}{\sqrt{(a^2 + b^2)}} \right] \dots (1).$$

$$\text{Put } \theta = \cot^{-1} b/a, \text{ then } \sin \theta = \frac{a}{\sqrt{(a^2 + b^2)}}, \text{ and } \cos \theta = \frac{b}{\sqrt{(a^2 + b^2)}}.$$

\therefore (1) reduces to

$$\frac{b - d/dx}{a^2 + b^2} \cos ax = (a^2 + b^2)^{-\frac{1}{2}} (\cos \theta \cos ax + \sin \theta \sin ax) = (a^2 + b^2)^{-\frac{1}{2}} \cos(ax - \theta).$$

$$\begin{aligned}
& \text{When } n=2, \frac{b-d/dx}{a^2+b^2} \left[\frac{1}{(a^2+b^2)^{\frac{1}{2}}} \cos(ax-\theta) \right] \\
&= \frac{1}{a^2+b^2} \left[\frac{b \cos(ax-\theta) + a}{\sqrt{a^2+b^2}} \sin(ax+\theta) \right] \\
&= \frac{1}{a^2+b^2} \left[\cos\theta \cos(ax-\theta) + \sin\theta \sin(ax-\theta) \right] = (a^2+b^2)^{-\frac{1}{2}} \cos(ax-2\theta).
\end{aligned}$$

$$\begin{aligned}
& \text{When } n=3, \therefore \frac{b-d/dx}{a^2+b^2} \left[\frac{1}{(a^2+b^2)^{\frac{1}{2}}} \cos(ax-\theta) \right] \\
&= \frac{1}{a^2+b^2} \left[\frac{b \cos(ax-2\theta) a \sin(ax-2\theta)}{\sqrt{a^2+b^2}} \right] = (a^2+b^2)^{-\frac{3}{2}} \cos(ax-3\theta).
\end{aligned}$$

This method holds for $(n-1)$ terms.

$$\begin{aligned}
& \text{Hence } y = e^{-bx} (c_1 + c_2 x + c_3 x^2 + c_4 x^3 + \dots c_n x^{n-1}) \\
& \quad + (a^2 + b^2)^{-\frac{1}{2}n} \cos[ax - n \cot^{-1}(b/a)].
\end{aligned}$$

SECOND SOLUTION.

$$\begin{aligned}
& \text{Put } D = \frac{d}{dx}, \text{ then we have } \frac{1}{(D+b)^n} \cos ax = \frac{(D-b)^n}{(D^2-b^2)^n} \cos ax \\
&= \frac{(-1)^n (b-D)^n}{(a^2+b^2)^n} \cos ax.
\end{aligned}$$

$$\begin{aligned}
& \text{The numerator} = (-1)^n \left[b^n \cos ax + n a b^{n-1} \sin ax \right] - \frac{n(n-1)}{2!} a^2 b^{n-2} \cos ax \dots \\
&= (-1)^n \cos[ax - n \cot^{-1}(b/a)]
\end{aligned}$$

The value of y is the same as in I.

Also solved by WILLIAM HOOVER, G. B. M. ZERR, and the PROPOSER.

Professor Landis remarks that the exponent of a^2+b^2 is given $\frac{1}{2}n$ in Johnson's *Differential Equations* (problem 17, page 122) instead of the correct result $-\frac{1}{2}n$.

122. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics in Drury College, Springfield, Mo.

$$\text{Solve the differential equation } (y-x)\sqrt{1+x^2} \frac{dy}{dx} = n(1+y^2)^{\frac{1}{2}}$$

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy, Ohio University, Athens, O.

$$\text{Let } x = \tan \theta, y = \tan \phi, \text{ then } \sin(\phi - \theta) d\phi = n d\theta \dots (1).$$